The Novel Proportionate Normalized Subband Adaptive Filter Algorithms for Sparse System Identification

Mohammad Shams Esfand Abadi and Sima Kadkhodazadeh

***Abstract*—System identification is one of the most important applications of adaptive filter algorithms. Some systems such as echo response that must be identified by the echo canceller have sparse nature. Classic adaptive filter algorithms like LMS or NLMS have poor performance in this way. Proportionate NLMS (PNLMS) algorithm has been developed to make improve poorly performance of NLMS in system identification, unfortunately it suffers from slow down during adaption process. To solve this problem, the concept of proportionate adaptation is extended to the normalized subband adaptive filter (NSAF), and three types proportionate normalized subband adaptive filter (PNSAF) algorithms are established in this paper. Proposed algorithms are proportionate normalized subband adaptive filter ++ (PNSAF++), the set-membership PNSAF (SM-PNSAF) and the set-membership PNSAF++ (SM-PNSAF++). Here we demonstrate that PNSAF++ algorithm improve the convergence rate of PNSAF in sparse channels. The SM-PNSAF and SM-PNSAF++ also exhibit good performance with significant reduction in the overall computational complexity compared with the ordinary PNSAF. The simulation results show good performance of the proposed algorithms.**

***Index Terms*—Proportionate normalized subband adaptive filter, set-membership, sparse system identification**

I. Introduction

Adaptive system identification is such an important problem especially when the system impulse response is sparse. Classic adaptive filter algorithms such as normalized least mean squares (NLMS) have low convergence rate in identification of sparse channel [1]-[3]. To solve this problem the proportionate adaptive filters have been proposed [4]. The basic principle of PNLMS is to adapt each coefficient with an adaptation gain proportional to its own magnitude. In this way this algorithm uses different step-sizes proportional to the estimated magnitude of the coefficients for each adaptive filter coefficient. PNLMS has much faster initial convergence than NLMS when the echo path is sparse. But unfortunately, it slows down after initial fast convergence, and also this algorithm has poor performance in dispersive channels. In [5], the PNLMS++ algorithm was proposed to alternate the PNLMS and NLMS algorithms during the adaptation. This strategy leads to fast initial convergence and at least as fast convergence as the NLMS algorithm later on. But PNLMS++ works well for only in the two extreme cases When the impulse response is sparse or highly dispersive To have fast convergence, low steady-state MSE, and low computational complexity at the same time, the set-membership adaptive filter algorithm like set-membership normalized LMS (SM-NLMS) has been proposed in [6].

In [7], the subband adaptive algorithm called normalized subband adaptive filter (NSAF) was developed based on a constrained optimization problem. The filter update equation proposed in [7] is similar to what proposed in [8] and [9], where the fullband filters are updated instead of subfilters as in the conventional SAF structure [10] and [11]. In this paper the concept of proportionate adaptation is extended to the normalized subband adaptive filter (NSAF), and three proportionate normalized subband adaptive filter algorithms are established. The proposed proportionate normalized subband adaptive filter (PNSAF) are PNSAF++, the set-membership PNSAF (SM-PNSAF) and the set-membership PNSAF++ (SM-PNSAF++).

This paper is organized as follows. In section II, we briefly review PNLMS, PNLMS++ and also set membership-PNLMS. In Section III, NSAF structure will be described and PNSAF algorithms are established. In Section IV we present several simulation results to show the good performance of the proposed algorithms. Finally, the conclusion of the paper is presented in Section V.

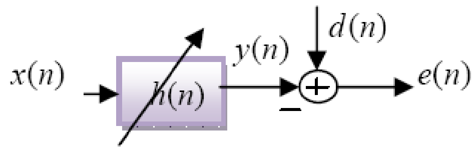

Fig. 1. Prototypical adaptive filter setup

II. Review of the NLMS, PNLMS ,PNLMS++ and Set Membership Algorithms

In Fig. 1 the prototype of adaptive filter setup illustrated, where $\mathbf{x}(n)$, $d(n)$ and $e(n)$ are the input, the desired and the output error signals, respectively. $\mathbf{h}(n)=[h_0(n),h_1(n),\ldots,h_{M-1}(n)]^T$ is $M+1$ the column vector of filter coefficients at time n.

The weight vector update equation for PNLMS is given by [4]:

$$\mathbf{h}(n+1)=\mathbf{h}(n)+\frac{\mu\mathbf{G}(n)}{\mathbf{x}^T(n)\mathbf{G}(n)\mathbf{x}(n)+\varepsilon}\mathbf{x}(n)e(n) \qquad (1)$$

where $e(n)=d(n)-\mathbf{x}^T(n)\mathbf{h}(n)$, ε is the regularization parameter, and μ is the step-size that determines the convergence speed and excess mean-square error (EMSE). $\mathbf{G}(n)=diag\{g_0(n),g_1(n),\ldots,g_{M-1}(n)\}$ is a diagonal matrix that adjusts the step-sizes of the individual taps of the

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filter. The diagonal elements of $G(n)$ are calculated from the following procedure [4]. Parameters δ and ρ are positive numbers with typical values $\delta = 0{:}01$, $\rho = 5/M$ [4].

The weight vector for PNLMS++ is similar to PNLMS and also NLMS. In fact this algorithm alternate the coefficient update between PNLMS and NLMS and gave similar performance to PNLMS for sparse systems but with better robustness in particular to echo path change [5].

Updating rules of PNLMS++ are based on switching between PNLMS and NLMS weight vector equations with n as index parameter. Consequently, PNLMS and NLMS are used alternately at odd and even sample instants n, respectively [5], [7].

As well as PNLMS, weight vector for SM-NLMS is given by [6]:

$$\mathbf{h}(n+1) = \mathbf{h}(n) + \frac{\alpha(n)}{\|\mathbf{x}(n)\|^2 + \varepsilon}\mathbf{x}(n)e(n) \tag{2}$$

where

$$\alpha(n) = \begin{cases} 1 - \dfrac{\gamma}{|e(n)|} & if\,|e(n)| > \gamma \\ 0 & otherwise \end{cases} . \tag{3}$$

III. Proportionate Normized SAF (PNSAF) Algorithms

In subband adaptive filtering (SAF), which proposed in [11] the input signal and desired response are band-partitioned into almost mutually exclusive subband signals. Based on principle of minimum disturbance in [1], novel design criterion for the SAF as a constrained optimization problem formulated in [12], [13] which involved multiple constraints imposed on the updated subband filter outputs. From one iteration to the next, these multiple subband constraints force each of the almost mutually exclusive subbands to converge almost independently without any influence from other subbands. Fig. 2 shows the structure of NSAF algorithm.

In this figure, $\mathbf{f}_0, \mathbf{f}_1, \ldots, \mathbf{f}_{N-1}$, are analysis filter unit pulse responses of an N channel orthogonal perfect reconstruction critically sampled filter bank system. $\mathbf{x}_i(n)$ and $d_i(n)$ are non subsample subband signals. It is important to note that n refers to the index of the original sequences and k denotes the index of the decimated sequences[1].

Similar to the NLMS algorithm, the weight vector of Normalized subband adaptive filters (NSAF) algorithms which had established in[13] is:

$$h(n+1) = h(n) + \mu \sum_{i=0}^{N-1} \frac{x_i(k)}{\|x_i(k)\|^2 + \varepsilon} e_{i,D}(k) \tag{4}$$

where

$$\mathbf{x}_i(k) = [x_i(kN), x_i(kN-1), \ldots, x_i(kN-M+1)]^T \tag{5}$$

$$d_{i,D}(k) = \mathbf{x}_i^T(k)\mathbf{h}(k+1) \quad for \quad i = 0, \ldots, N-1 \tag{6}$$

The $e_{i,D}(k) = d_{i,D}(k) - \mathbf{x}_i^T(k)\mathbf{h}(k)$ is the decimated subband error signal, and μ is chosen in the range $0 < \mu < 2$ [13]. The value of this parameter determines the convergence speed and the steady-state MSE of the NSAF..

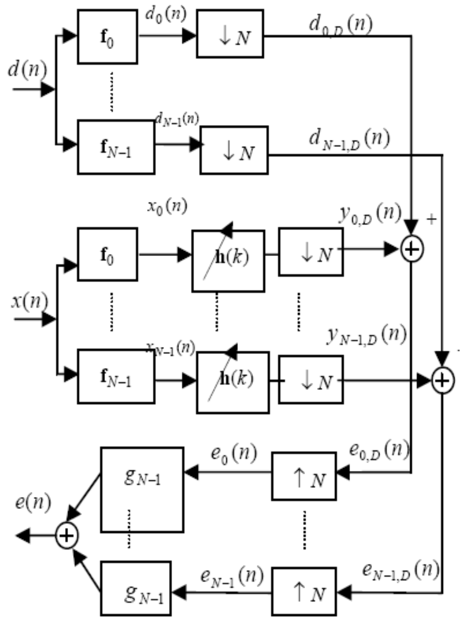

Fig. 2. Structure of NSAF

The weight vector update equation in PNSAF can be stated as[13]

$$\mathbf{h}(n+1) = \mathbf{h}(n) + \mu \sum_{i=0}^{N-1} \frac{\mathbf{P}(k)\mathbf{x}_i(k)}{\mathbf{x}_i^T(k)\mathbf{P}(k)\mathbf{x}_i(k) + \varepsilon} e_{i,D}(k) \tag{7}$$

where $\mathbf{P}(k) = diag\{p_0(k), p_1(k), \ldots, p_{M-1}(k)\}$ is a diagonal matrix and the diagonal elements of $P(k)$ are calculated from the following procedure [4]:

$$\beta_m(n) = \max\{\rho \max\{\delta, |h_0(n)|, \ldots, |h_{M-1}(n)|\}, |h_m(n)|\} \tag{8}$$

The diagonal elements of $P(k)$ are given by:

$$p_m(n) = \frac{\beta_m(n)}{\frac{1}{M}\sum_{i=0}^{M-1} \beta_i(n)} \tag{9}$$

In the PNSAF algorithm, different step-sizes is assigned to the coefficients based on their current estimated magnitudes in each subband..

PNSAF updating applies accordingly high adaptation gain to these coefficients, causing them to converge quickly, thereby obtaining a fast initial reduction in error. A low adaptation gain is applied by PNSAF to the coefficients in the inactive regions, because of their low amplitude, in order that the final misadjustment of the proportionate scheme is no worse than comparable non-proportionate schemes. Therefore if the current magnitude of a coefficient is large, large step-size will be assigned, and vice versa [14].

[1] It means that in the SAF, the filter vector update is performed each time N new samples have entered the system.

A. Algorithm II: Proportinate NSAF ++(PNSAF++)

Based on [5], PNLMS++ is an adaptive algorithm which alternate the coefficient update between PNLMS and NLMS. correspondingly, this concept could be extended to proportionate NSAF structure and the weight vector update expected to alternate between PNSAF and NSAF updating equation. Based on this idea, we propose a new algorithm that uses equations (7) and (4) at same time as it's update rule. This algorithm that we called it PNSAF++ , switches between PNSAF and NSAF. When n takes odd value, algorithm will switch to NSAF and similarly algorithm will switch to PNSAF if n takes even. Simulations in section IV show that PNSAF++ improves the convergence rate of ordinary PNSAF and also NSAF adaptive algorithms.

B. Algorithm III: Set Membership PNSAF(SM-PNSAF)

The SM-NSAF that established in [14] subject to

$$\mathbf{h}(k) \in \left(\psi_{k,0} \cap \psi_{k,1} \cap \ldots \cap \psi_{k,N-1}\right) \tag{10}$$

where

$$\psi_{k,i} = \left\{\mathbf{h} \in \mathbf{R}^M : \left|d_{i,D}(k) - \mathbf{x}_i^T(k)\mathbf{h}\right| \le \gamma\right\} \tag{11}$$

This aim is obtained by an orthogonal projection of the previous estimate of h onto the closest boundary of $\psi_{k,i}$ in each subband. Doing this, the filter vector update equation for SM-NSAF stated in [14] is given by

$$\mathbf{h}(n+1) = \mathbf{h}(n) + \mu \sum_{i=0}^{N-1} \alpha_i(k) \frac{\mathbf{x}_i(k)}{\mathbf{x}_i^T(k)\mathbf{x}_i(k) + \varepsilon} e_{i,D}(k) \tag{12}$$

where

$$\alpha_i(n) = \begin{cases} 1 - \dfrac{\gamma}{\left|e_{i,D}(n)\right|} & \text{if } \left|e_{i,D}(n)\right| > \gamma \\ 0 & \text{otherwise} \end{cases} \tag{13}$$

The same as SM-PNLMS ,the filter vector update equation for SM-PNSAF can be stated as

$$\mathbf{h}(n+1) = \mathbf{h}(n) + \mu \sum_{i=0}^{N-1} \alpha_i(k) \frac{\mathbf{P}(k)\mathbf{x}_i(k)}{\mathbf{x}_i^T(k)\mathbf{P}(k)\mathbf{x}_i(k) + \varepsilon} e_{i,D}(k) \tag{14}$$

In the proportionate algorithm, $\mathbf{P}(k)$ will be a diagonal matrix. Based on [14], to make a good choice for $\mathbf{P}(k)$, the goal is to reduce the length of the initial transient for estimating the dominant peaks in the impulse response and, thereafter, to emphasize the input-signal direction to avoid a slow second transient. Furthermore, the solution should not be sensitive to the assumption of a sparse system. Refer to [14], as $\alpha(k)$ is a good indicator of how close a steady-state solution is, the proposed $P(k)$ is

$$p_i(k) = \frac{1-\kappa\alpha_i(k)}{N} + \frac{\kappa\alpha_i(k)\left|h_i(k)\right|}{\left\|\mathbf{h}(k)\right\|_1} \tag{15}$$

where $\kappa \in [0, 1]$ and $\left\|\mathbf{h}(k)\right\|_1 = \sum_{i=0}^{N-1}\left|h_i(k)\right|$. The constant κ is included to increase the robustness for estimation errors in $\mathbf{h}(n)$, and from the simulations provided in Section IV, $\kappa = 0.5$ shows good performance for both sparse and dispersive systems. For κ = 1, the algorithm will converge faster but will be more sensitive to the sparse assumption.

C. Algoithm IV: Set Membership PNSAF++ (SM-PNSAF++)

Similar to SM-PNSAF, if set membership concept is extended to PNSAF++, the new algorithm which is called SM-PNSAF++ will be established. The Proposed algorithm switches between equations (19) and (21) with index n when SM-PNSAF and SM-NSAF are used alternately at odd and even sample instants, respectively. Simulations in section IV show that this algorithm has similar convergence rate to SM-PNSAF.

IV. Simulation Results

We demonstrate the performance of the proposed algorithms by several computer simulations in a system identification scenario. Two impulse response of the unknown systems are shown in Fig. 3. Fig 3 (a) show sparse impulse response that consisted of an M = 100 truncated FIR model from a digital microwave radio channel [15][2]. Fig. 3 (b) shows the dispersive impulse response that has been generated at random. The input signal, $\mathbf{x}(n)$ is a first order autoregressive (AR) signal generated according to

$$x(n) = \tau\, x(n-1) - w(n) \tag{16}$$

where $w(n)$ is zero mean white Gaussian signal. The parameter τ was set to 0.7. The measurement noise, $v(n)$ with $\sigma_v^2 = 10^{-3}$ was added to the noise free desired signal generated through $d(n) = \mathbf{h}_t^T\mathbf{x}(n)$, where $\mathbf{h}_t$ is the true unknown filter vector. The filter bank used in the subband adaptive filters was the four subband Extended Lapped Transform (ELT) [15]. In all the simulations, the simulated learning curves, obtained by ensemble averaging over 200 independent trials. Also the value of γ was set to $\sqrt{5\sigma_v^2}$ [13].

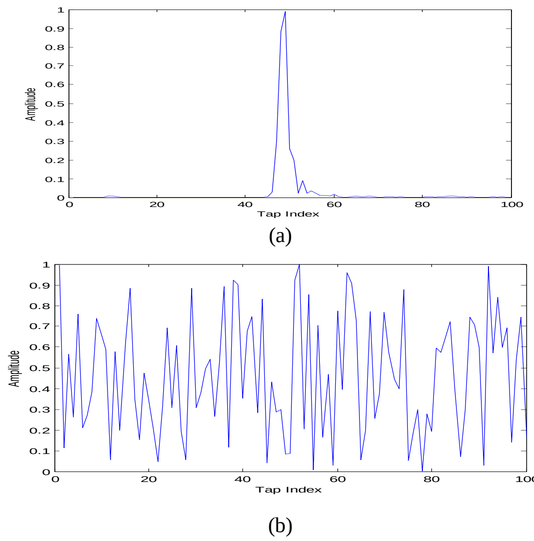

Fig. 3. Two different impulse response. (a) sparse (b) dispersive

[2] The coefficients of this complex-valued baseband channel model can be downloaded from http://spib.rice.edu/spib/microwave.html

We set the parameters to $\mu = .5$, $\delta = 0.01$ $\rho = 0.05$. Figs. 4 and 5 show the learning curves of PNSAF and PNSAF++ algorithms for different impulse responses in Fig. 3.

Fig. 4 shows the learning curves when the impulse response of Fig. 3 (a) is identified. As we can see the PNSAF and PNSAF++ algorithms have about same convergence rate but PNSAF is a little faster than PNSAF++ which is not considerable.

Fig. 5 shows the learning curve when dispersive response of Fig. 3 (b) is identified. We can easily understand that PNSAF++ has better performance and also faster convergence rate than simple PNSAF. This result is equal to what is explained in section III. Figure 6 and 7 show the learning curve of SM-PNSAF and SM-PNSAF++ for different impulse response that shown in figure (3).

Fig. 6 illustrates the learning curve of SM-PNSAF and SM-PNSAF++ when sparse response of Fig. 3 (a) is identified. It shows that SM-PNSAF has faster convergence rate than SM-PNSAF++ although there is no significant difference between them same as what achieved before.

Fig. 7 exhibits the learning curve of SM-PNSAF and SM-PNSAF++ when dispersive response of Fig. 3 (b) is identified. Corresponds to what's established in proposed algorithms, SM-PNSAF and SM-PNSAF++, in section III, for dispersive channel, SM-PNSAF++ is faster than SM-PNSAF.

At last, all proposed algorithms have been comprised in figure 7 and 8 for different impulse responses in figure 3.

Fig. 8 shows PNSAF, PNSAF++, SM-PNSAF and SM-PNSAF++ when sparse response of Fig. 3 (a) is identified. It could be understand that PNSAF and PNSAF++ have more convergence rate in sparse channel than SM-PNSAF and SM-PNSAF++ although based on the results which is shown in table I, the average numbers of updates for each subands in SM-PNSAF are 170,143,138 and 112 respectively instead of 1000 for each suband in PNSAF algorithm. The average numbers of updates for each subands of SM-PNSAF++ are 243,230,209 and 193 respectively instead of 1000 for each suband IN PNSAF++ algorithm refers to the results of table I. This result proves that set membership proportionate SAF algorithms such as SM-PNSAF and SM-PNSAF++ ultimately reduces computational complexity and also much faster than simple PNSAF algorithms like PNSAF and PNSAF++ during updating process .

Fig. 9 shows learning curve of PNSAF, PNFSA++, SM-PNSAF and SM-PNSAF++ in dispersive channel that shown in figure 3.(b). we can see SM-PNSAF++ and PNSAF++ has faster convergence rate in dispersive channels than PNSAF and SM-PNSAF. This figure also illustrates that in dispersive channel PNSAF++ has better performance than SM-PNSAF++ . Based on the results of table I for dispersive channel, The average numbers of updates for each subands of SM-PNSAF++ are 287,279,265 and 257 respectively instead of 1000 for each suband IN PNSAF++. These results are achieved for SM-PNSAF in dispersive channel too. Refers to the results of table I, the average numbers of updates for each subands of SM-PNSAF in dispersive channel are 219,182,174 and 168 respectively instead of 1000 for each suband in PNSAF. It proves that SM-PNSAF++ and SM-PNSAF highly reduce computational complexity of adaptive algorithms and there are also much faster than PNSAF++ and PNSAF respectively.

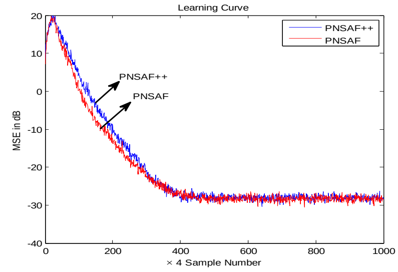

Fig. 4. Learning curve of PNSAF and PNSAF++ for sparse channel in fig 3.(a)

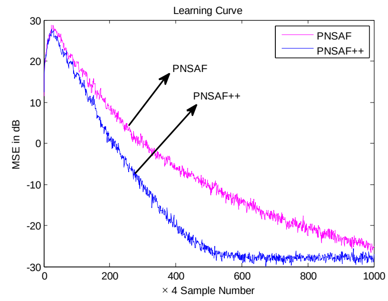

Fig. 5. Learning curve of PNSAF and PNSAF++ for dispersive channel in fig 3.(b)

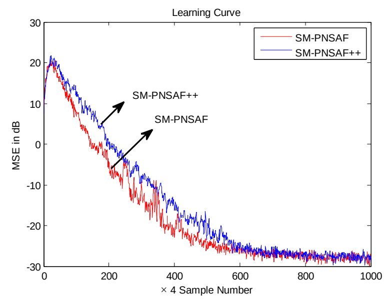

Fig. 6. Learning curve of SM-PNSAF and SM-PNSAF++ for sparse channel in fig 3.(a)

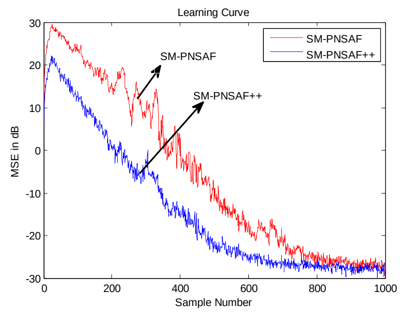

Fig. 7. Learning curve of SM-PNSAF and SM-PNSAF++ for dispersive channel in fig 3.(b)

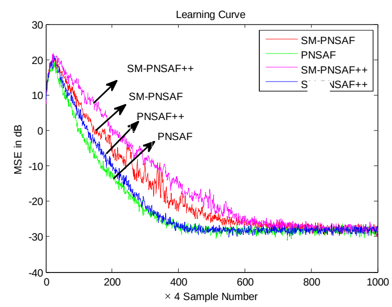

Fig. 8. Learning curve of PNSAF, PNSAF++, SM-PNSAF and SM-PNSAF++ for sparse channel in fig 3.(a)

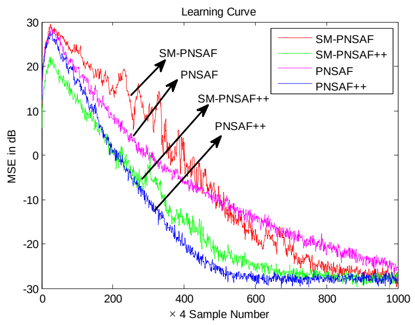

Fig. 9. Learning curve of PNSAF, PNSAF++, SM-PNSAF and SM-PNSAF++ for dispersive channel in fig 3.(b)

TABLE I: AVERAGE NUMBER OF UPDATES FOR EACH ALGORITHM

		Average Number of Updates for each subbands			
Algorithms	***Channel type***	***Subband 1***	***Subband 2***	***Subband 3***	***Subband4***
SM-PNSAF	spares	170	143	138	112
	dispersive	219	182	174	168
SM-PNSAF ++	spares	243	230	209	193
	dispersive	287	279	265	257
PNSAF and PNSAF++	spares	1000	1000	1000	1000
	dispersive	1000	1000	1000	1000
PNSAF++	spares	1000	1000	1000	1000
	dispersive	1000	1000	1000	1000

V. CONCLUSION

In this paper, the concept of proportionate adaptation was extended to the NSAF and the family of proportionate normalized subband adaptive filter algorithms was established. The proposed algorithms are suitable for sparse system identification. The PNSAF algorithm had initial fast convergence in sparse channel but PNSAF++ had much faster than it in dispersive channel. We also established SM-PNSAF and SM-PNSAF++. The SM-PNSAF exhibited good performance with significant reduction in the overall computational complexity but in comparison to PNSAF in sparse channel has fewer convergence rates. We demonstrated the proposed algorithms through several simulation results.

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